

# Regression Discontinuity Designs

Statistic Modeling & Causal Inference | Oswald & Ramirez-Ruiz

# Agenda

- Lecture Review
  - Basic idea behind RDD
  - Continuity of potential outcomes
  - Falsification Checks
  
- RDD in R

# Example case

- Effects of alcohol consumption (**treatment**) on mortality (**outcome**)
  - Carpenter & Dobkin (2009)
- **Running variable:** Age
- **Cut-off:** Minimum Drinking Age



# Core Idea

Drinking

Drinking Age

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- **Treatment** assigned according to a **rule** based on another variable (**running or forcing variable**)
- Treated and control units may differ in their potential outcomes based on the forcing variable (non-random selection into treatment)
- However, whether units end up just below or just above the threshold can be assumed as a matter of chance (local randomization)
- Units around the cutoff are assumed to be similar in every way except the treatment assignment
- (Local) treatment effect can be determined by comparing cases on both sides of the cut-off

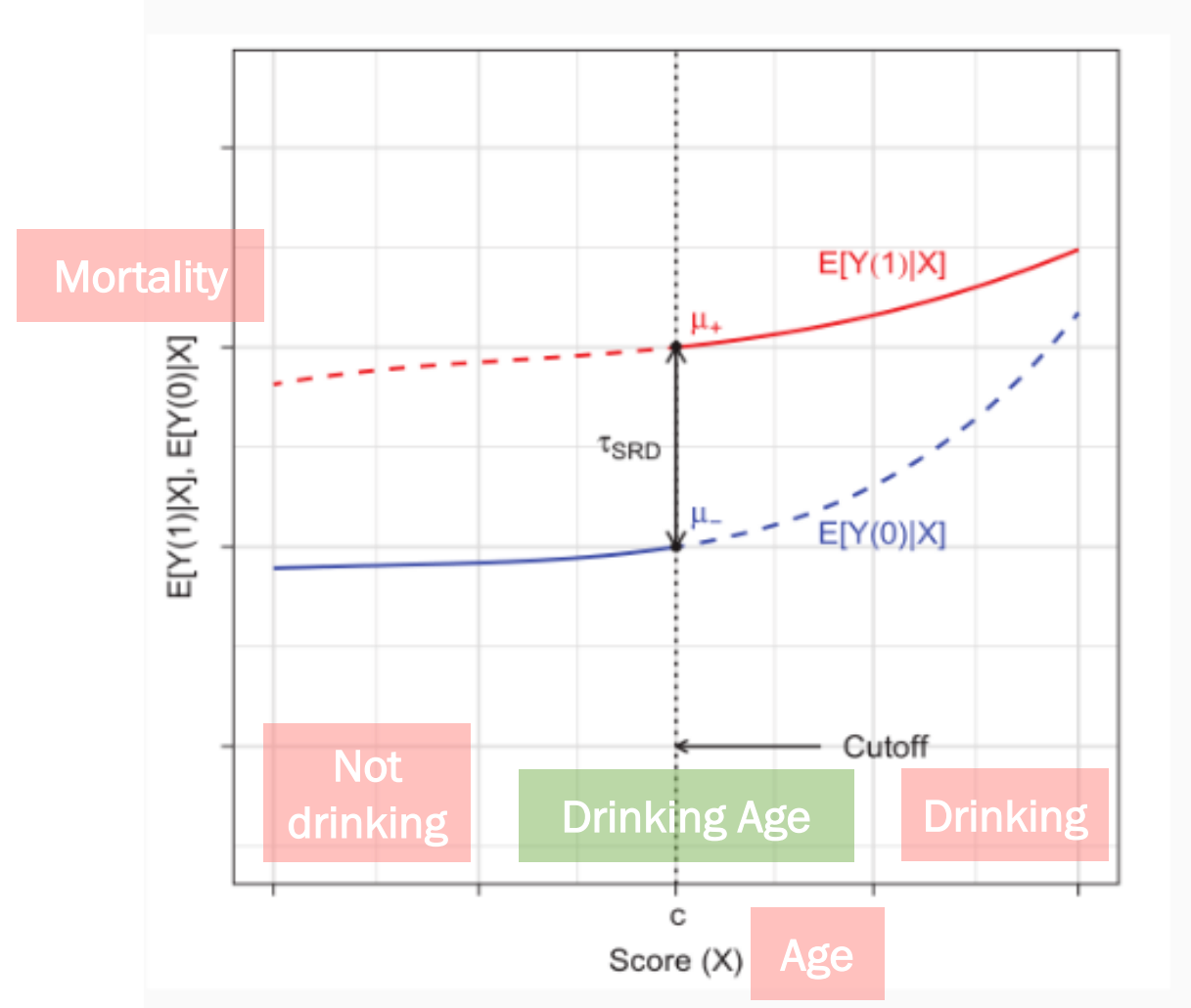
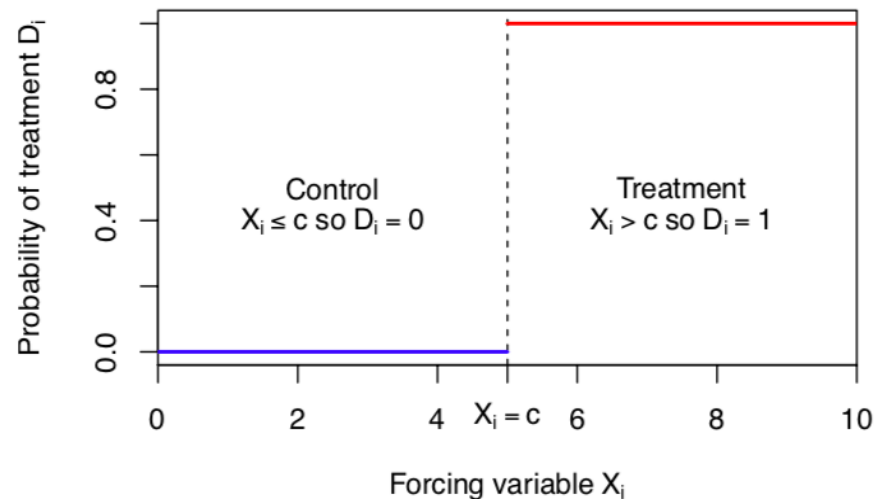
Age

Old & drinking

18 - 2 days /  
18 + 2 days

# Sharp RDD

- Forcing variable (X) **perfectly determines** which side of the cut-off people are (treatment or control)
- We can only estimate the effect at a **single point**: the cutoff or threshold




# Key Assumption

- **Continuity of average potential outcomes** (on both sides of the cut-off)
  - units on one side of the threshold have essentially the same potential outcomes from those just on the other side
- This allows us to do a tiny bit of extrapolation and estimate **LATE** at the threshold
- BUT: this assumption can easily be violated:
  - For example, by some other variable driving differences at the cut-off

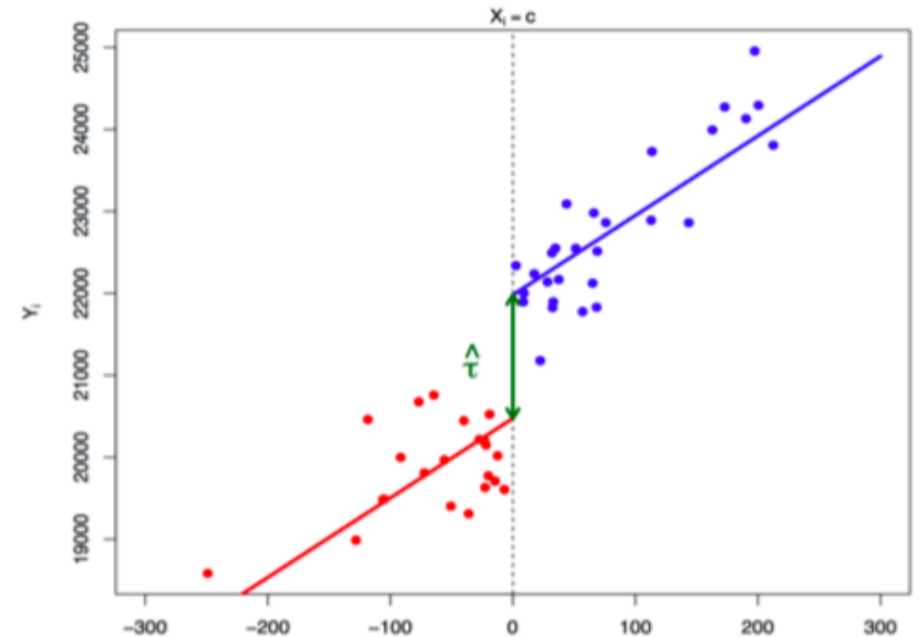
# Estimating LATE (local polynomial approach)

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- Decide which **model** is the most appropriate given the nature of the data: linear with a common slope, linear with different slopes, or nonlinear.
- Choose a **kernel** function for weighting the observations close to cutoff. (common practice: triangular)
- Choose a window or **bandwidth** ( $h$ ) around the threshold ( $c$ ) to create a “discontinuity sample.”
- The narrower the better, but can you afford losing many observations? (bias-variance tradeoff)
- Recode **forcing variable**  $X$  to deviations from threshold (centered on 0).
- Fit the (WLS) regression model for the observations, within the window, **above** the cutoff.
- Fit the (WLS) regression model for the observations, within the window, **below** the cutoff.
- **The local average treatment effect is the difference between the two intercepts at the cutoff.** 

# Linear with a Common Slope

- Assumptions:
  - Potential outcomes under treatment and under control are linear in  $X$
  - Treatment effect does not depend on the value of  $X_i$ . The effect is constant along  $X_i$ .
- In this case, we regress the observed outcome  $Y_i$  on  $D_i +$  centered  $X_i$ .



Mortality  $\tau$  18 Age

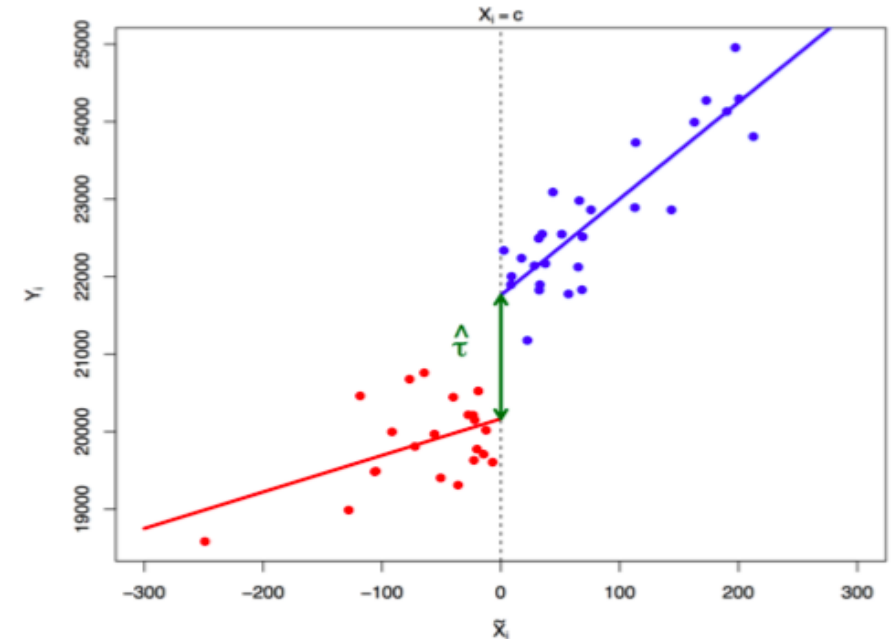
$$\text{Model is } Y_i = \beta_0 + \tau D_i + \beta_1 X_i + \epsilon_i$$

$$\begin{cases} <18 \\ >18 \end{cases} \begin{cases} E[Y_{0i}|X_i] = \beta_0 + \beta_1 * X_i \\ E[Y_{1i}|X_i] = \beta_0 + \tau + \beta_1 * X_i \end{cases}$$



# Linear with Different Slopes

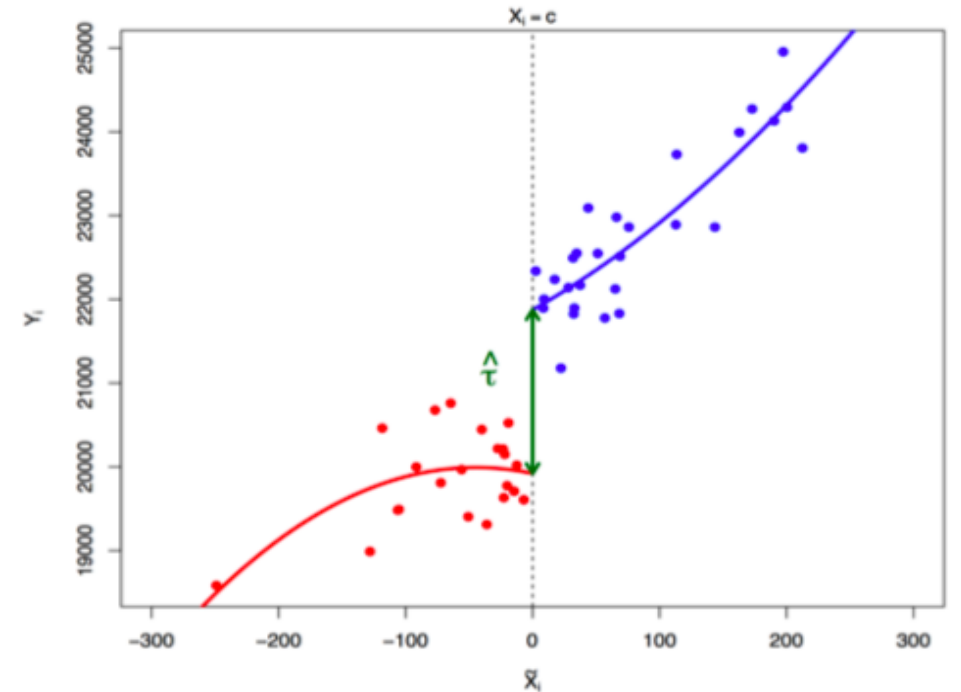
- Assumptions:
  - Potential outcomes under treatment and under control are linear in  $X$
  - Treatment effect can vary for different values of  $X_i$ .
- In this case, we regress the observed outcome  $Y_i$  on the interaction  $D_i * X_i$ .



Mortality	</> 18	Age	Interaction	<18	{	$E[Y_{0i} X_i] = \beta_0 + \beta_1 * X_i$
Model is $Y_i = \beta_0 + \tau D_i + \beta_1 X_i + \phi D_i X_i + \epsilon_i$						>18

# Non-linear

- Assumptions:
  - Potential outcomes are allowed to be non-linear in  $X$  but must be correctly specified
  - Treatment effect can vary for different values of  $X_i$ .
- Model can include quadratic, cubic, etc. terms in  $X_i$  and their interactions with  $D_i$  in the equation.



! Be cautious about high-order polynomials: they are difficult to fit, make lots of assumptions about the data, and are sensitive to outliers.

Polynomial

New Interaction

$$\text{Model: } Y_i = \beta_0 + \tau D_i + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i D_i + \beta_4 X_i^2 D_i + \epsilon_i$$

# And how do I specify my model? 🙄

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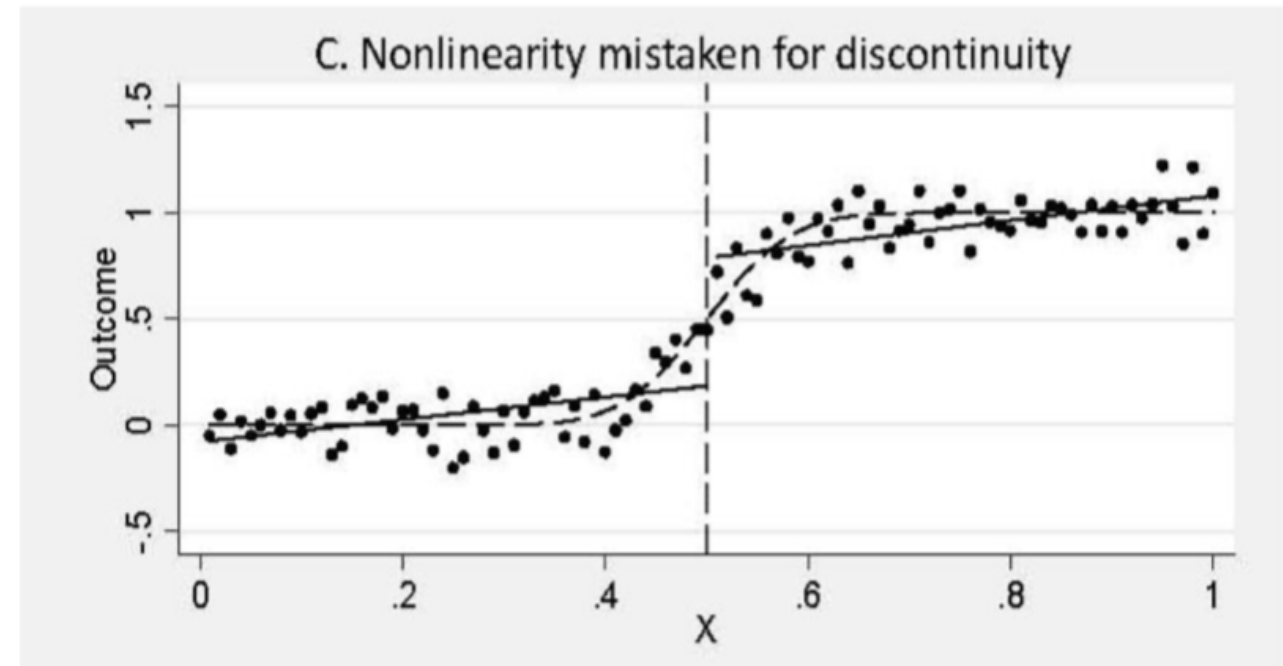
- Model specification is a trade-off between **bias** and **variance**
  - If you choose nonlinear, you might reduce variance because you can pick up every sensitivity in the data, but estimates will be biased due to following “noise.”
- Standard practice: Try and **compare different specifications** to show robustness
  - Ideally you are looking for similar results across different models.
- Always start with a visual inspection: see scatterplot and run a local regression (such as LOWESS) to guide choice
- Remember each model corresponds to a particular set of assumptions about the POs.

# Falsification Checks

## Sensitivity:

Are results sensitive to alternative specifications?

- Nonlinear relation  $\neq$  discontinuity
- If units start curving up near lower threshold and down near upper, it might just be non-linearity vs. a discontinuity jump.

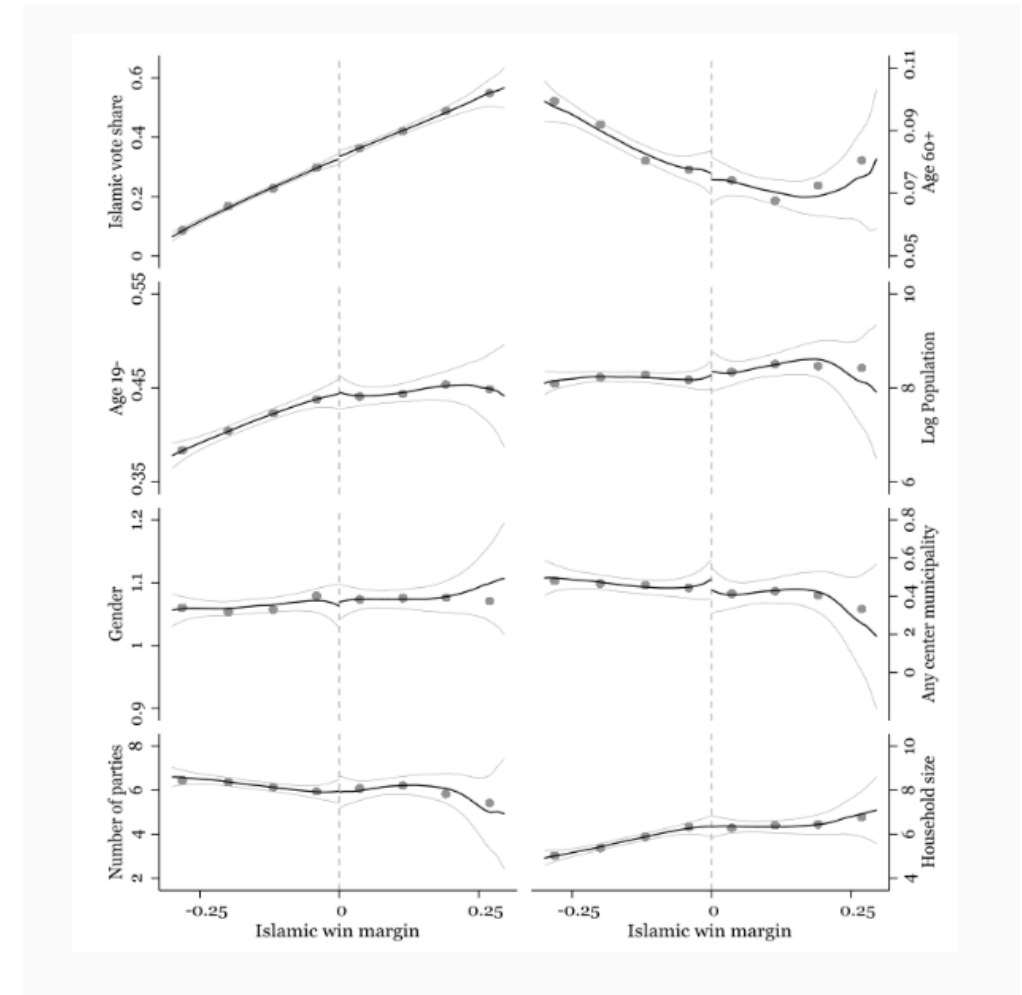


# Falsification Checks

## Balance checks:

Does any covariate  $Z_i$  jump at the threshold?

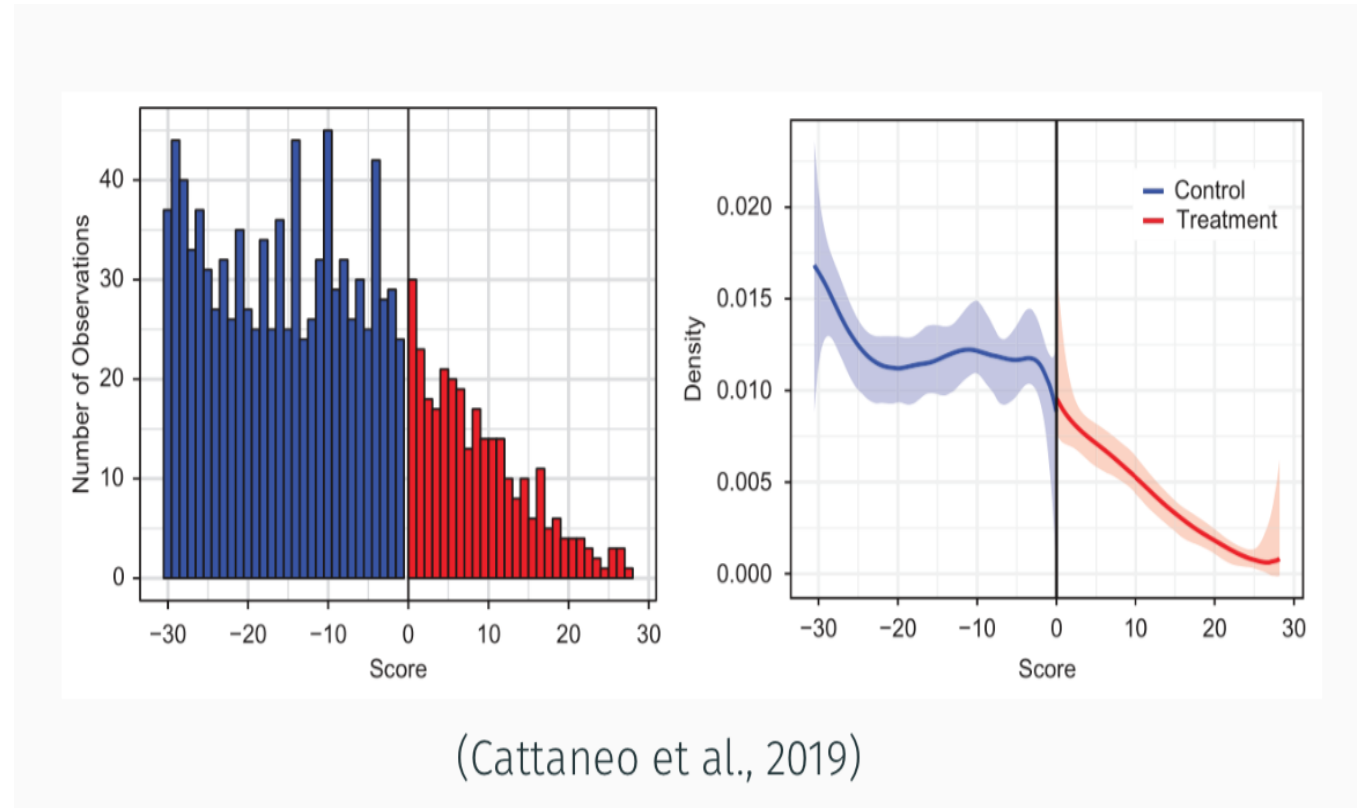
- Aiming for a scenario where individuals are pretty much identical except for treatment ‘assignment’.
- We should only see a jump in  $Y$ , not on other **pre-treatment** or **post-treatment** (not affected by treatment) variables.



# Falsification Checks

## Sorting:

- Do units sort around the threshold? Is there a jump in number of observations around the cut-off?
- Sometimes there is an incentive to end up above or below a threshold. An agent's behavior can invalidate the continuity assumption. Local randomization would not hold.

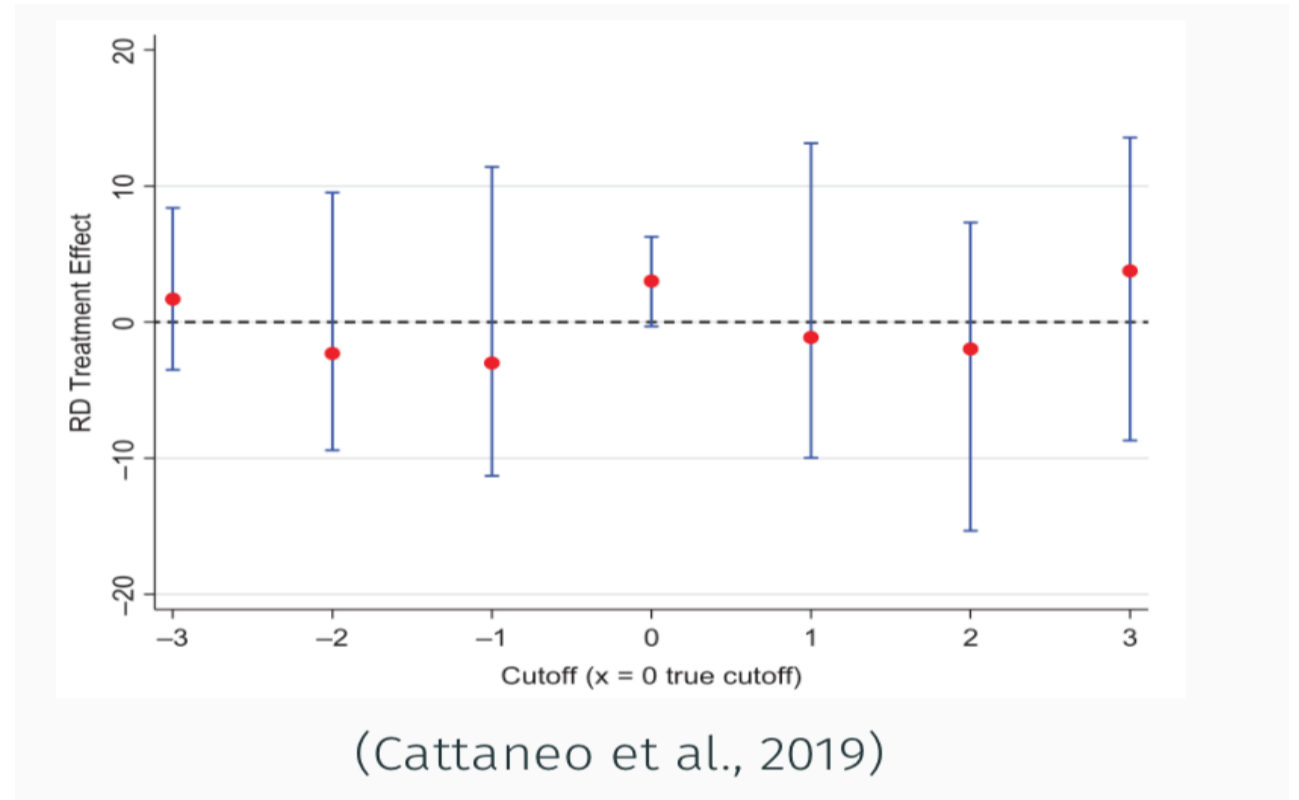


# Falsification Checks

## Artificial cut-off values:

Do jumps occur at placebo thresholds?

- If they do, this could mean something else is going on that could challenge our research design.



# Falsification Checks

## Sensitivity to cases near cutoff:

Do results change if we exclude cases near the threshold?

- Remember the different weights in the kernel definition.
- If self selection into treatment took place, the units closest to the cutoff would be the most likely units to engage in it.

## Sensitivity to bandwidth choice:

Do results change if we specify the bandwidth differently?



# Further Resources

For any coding issues – [Stackoverflow](#)

Hertie's Data Science Lab – [Research Consulting](#)