

Instrumental Variables

Statistic Modeling & Causal Inference – Oswald | Ramirez-Ruiz

Agenda

- Lecture review
- ITT and LATE
- Assumptions for IV designs

- Estimating LATE in R

Motivation

Experiments

- It is often not possible to force subjects to **comply** with their assignment to treatment/control.
- Those who choose to take the treatment may **systematically differ** from those who do not (selection bias)

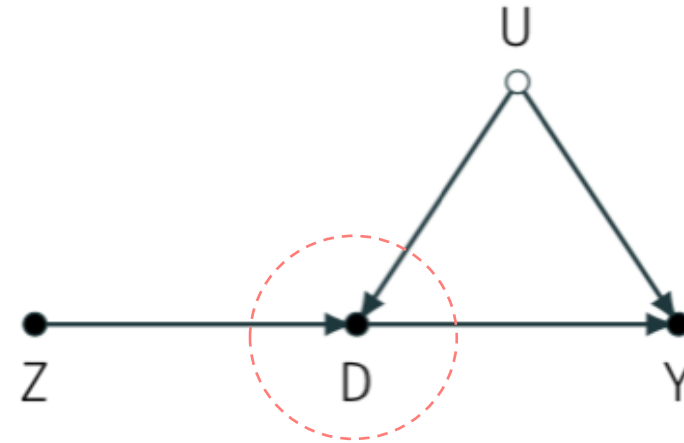
Observational Studies

- Often the relationships between our variables of interest are affected by confounders that are **unmeasured or unmeasurable**.
- We can exploit **natural experiments** that generate (as if) random variation in our independent variable in order to estimate causal effects.

Instrumental Variable

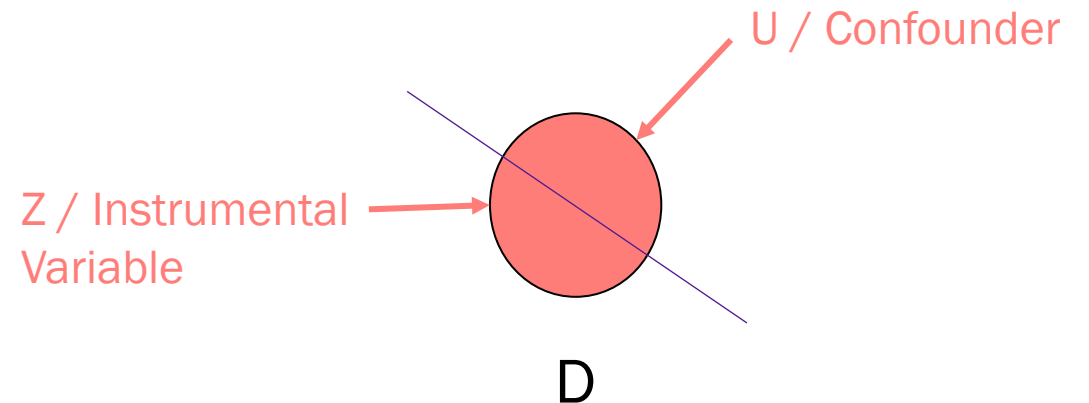
General Idea

- In order to bypass the problems of non-compliance and confounders, use an exogenous variable (Z) that affects the treatment or independent variable (D) but is **not** affected by confounders (U).



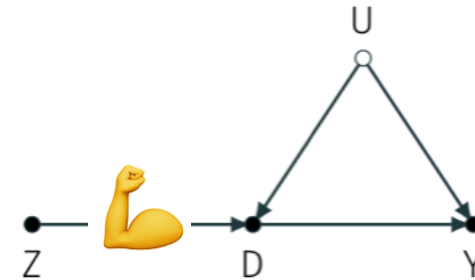
Practical Procedure

- We split the variation of D into two parts:
- One potentially related to confounders (U), observed or unobserved.
- One truly **exogenous**

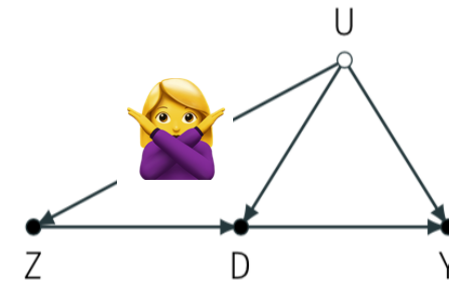


Requirements for a valid IV

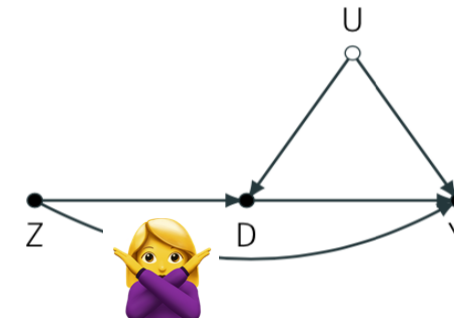
1. Z must indeed affect D (**relevant**)



2. Z must be **exogenous**, not a descendent of confounders. It must be as if randomly assigned.



3. Z must only affect Y through D (**exclusion restriction**)



Intent to Treat Effect (ITT)

- ITT is the effect of the instrumental variable itself on the outcome, regardless of actual treatment. It only considers *assignment* to the treatment or control groups.
- Because Z is randomized, ITT is identified by the difference in means of the outcome of interest between those assigned to treatment and those not.

$$ITT = E(Y_i | Z_i = 1) - E(Y_i | Z_i = 0)$$

Compliance Types

- Some people will always take the treatment, regardless of whether they are in treatment or control (**always-takers**),
- and some never will (**never-takers**).
- Some will always do as their told (**compliers**),
- and some will always do the opposite (**defiers**).

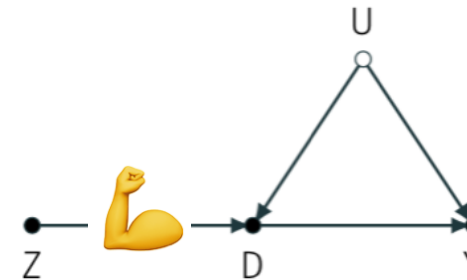
	$Z_i = 0$	$Z_i = 1$
$D_i = 0$	Complier/Never-taker	Defier/Never-taker
$D_i = 1$	Defier/Always-taker	Complier/Always-taker

We cannot directly identify the group to which any particular respondent belongs.

Formal IV Assumptions

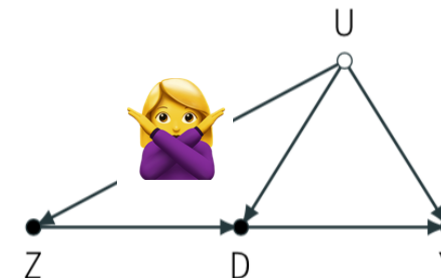
Relevance or nonzero average encouragement effect.

- Encouragement needs to make a difference.
- **Testable:** observe differences between treatment and control groups (**first stage**)



Exogeneity or ignorability of the instrument.

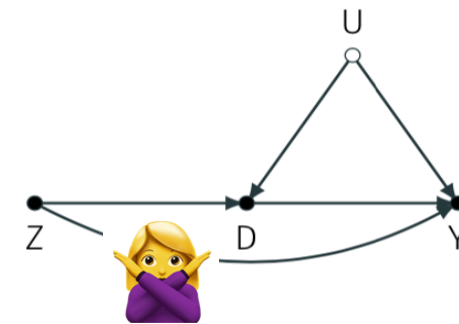
- Hypothetical potential outcomes must be independent from Z.
- Given by quasi-randomization of encouragement. Not empirically testable. A matter of plausibility.



Formal IV Assumptions

Exclusion restriction: instrument affects outcome **only** via treatment.

- Implies zero ITT effect for always-takers/never-takers.
- Hardly testable! (placebo/falsification criterion)



Monotonicity: effect of treatment is only in one direction.

- Implies we assume there are no defiers.
- Also hardly testable; matter of plausibility

	$Z_i = 0$	$Z_i = 1$
$D_i = 0$	Complier/Never-taker	[purple woman emoji]r/ Never-taker
$D_i = 1$	D [purple woman emoji]r/Always-taker	Complier/Always-taker

SUTVA: no spillovers

Homogeneity: constant treatment effect assumption

LATE

Local Average Treatment Effect (here with binary D and Y)

ITT can be decomposed into different subgroups:

$$ITT = \text{The intent to treat for compliers} + ITT \text{ for always-takers} + \\ ITT \text{ for never-takers} + ITT \text{ for defiers}$$

Under the **monotonicity** and **exclusion restriction**, this simplifies to:

$$ITT = ITT_{\text{compliers}} \times Pr(\text{compliers})$$

→ $ITT_{\text{compliers}}$ can then be interpreted as LATE:

$$ITT_c = \frac{ITT}{Pr(\text{compliers})} = LATE$$

Estimating LATE

”By hand”

Using the Wald estimator

$$\text{LATE} = \frac{\text{Cov}(Y_i, Z_i)}{\text{Cov}(D_i, Z_i)}$$

$$= \frac{E(Y_i|Z_i = 1) - E(Y_i|Z_i = 0)}{E(D_i|Z_i = 1) - E(D_i|Z_i = 0)}$$

(Be careful to weight the expected outcomes with the observed number of observations)

or

Two-stage least squares (2SLS)

A sequence of two regressions

1. **First stage:** Regress treatment D on instrument Z → $D = \gamma_0 + \gamma_1 Z_i + v_i$

Calculate predicted values (D-hat) of first stage regression.

2. Regress outcome Y on predicted values (D-hat). → $Y_i = \beta_0 + \beta^{2sls} \hat{D} + u_i$

The regression coefficient of D-hat is the LATE estimator.

It only retains the variation in D that is generated by (as if)

random variation in Z: the portion of the variation in D that

we wanted to isolate!

Further Resources

For any coding issues – [Stackoverflow](#)

Hertie's Data Science Lab – [Research Consulting](#)